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Common Factor Analysis versus Component Analysis: Some Well and Little Known Facts

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Component and common factor analyses are often selected as much because of one's underlying epistemology or paradigm (Gorsuch, 1988) as because of knowledge, but we do know considerably more about these two techniques now than we did twenty years ago. This article will show why common factor analysis is a highly desirable alternative to component analysis from primarily a knowledge-oriented perspective.

Because this is an invited article and is limited by the editor in length, I shall reference sources with greater detail (principal Gorsuch, 1974, 1983, 1988; also Cattell, 1978) and urge readers not convinced by this presentation to pursue the details for themselves. I also assume that different paradigms and purposes will indeed lead to different factor analyses, including occasionally component analysis (Gorsuch, 1983, chapter 17; 1988).

Two topics will not be considered per se. First, the rules for determining the number of the factors are the same for components as for common factor analysis, and the one I currently use is Velicer's MAP (for lack of space I shall forego the opportunity to show how the logic of MAP can be common factor based). Second, the effect of overextraction should be of lesser importance now that MAP exists, and so will not be discussed. This article shall consider seven salient issues and then shall place the current discussions in historical context for better understanding of why we are where we are.

Salient Issues

1. Common factor analysis is the general case of which component analysis is a special case. Common factor analysis includes variables with error and variables without error because, in the latter case, certain elements become zeros. Component analysis limits this broader model by an additional assumption: the variables are reproduced without error (i.e., without uniquenesses or residuals). (To see the evidence for this, compare equation 3.5.1 with 3.4.2 in Gorsuch, 1983, where the equations are developed so this is obvious). And
because component analysis is a special case of common factor analysis, a common factor analysis procedure does indeed produce components if they exist but the opposite is not necessarily true. In selecting a model, ask if your variables are truly measured without error; if so, component analysis is a possibility.

2. Some argue that it makes no difference whether components or common factor analysis is used. That is true for many situations (Gorsuch, 1974, 1983, 1988). However, there are other situations where results are obviously different. Gorsuch (1988) provides an example based upon, unfortunately, a real life incident. I had difficulty dissuading an experienced investigator from interpreting the component loadings because they looked high and obviously significant; I felt this was inappropriate because there was not one significant coefficient in the entire correlation matrix. A common factor analysis showed trivial loadings which were obviously insignificant.

Of course, components and common factors are the same if the communalities are high and the number of variables are high (e.g., greater than 30 according to Gorsuch, 1974, 1983, but see below for revision). But if common factor analysis produces more sensible results than component analysis in some cases and produces the same in other cases, there seems little advantage to recommending the special case component analysis over the general case common factor analysis. A parallel is the relationship of the critical ratio test to the $t$ test. The critical ratio works for large $Ns$ but not for small $Ns$; $t$ works for both. Over time, $t$ has replaced the easier-to-use critical ratio because it is more generally applicable.

3. Snook and Gorsuch (1989) investigated the degree to which components and common factors reproduced the original factor loadings in a Monte Carlo analysis. The conclusion was that common factor analysis produced results closer to the population values than component analysis, particularly with small numbers of variables and low to moderate communalities. Actually the study was mis-designed in one respect: we took seriously my earlier recommendation that there would be no difference with more than 30 variables. However, our greatest number of variables, 36, still found significant differences in favor of common factor analysis.

There are two reasons why Snook and Gorsuch (1989) differ from studies by Velicer and associates. First, Snook and Gorsuch used the original population factor loadings as the values to be reproduced, and compared both common factors and components to those population values. Velicer and associates (Velicer, personal communication) used, instead of the population loadings, the component loadings from a component analysis of the population correlations...
generated by the population loadings. (Snook and Gorsuch would have followed a similar procedure if they had, instead of comparing to the original population loadings, compared values to a common factor analysis of the population correlations using the same communality and extraction procedures that they would later test.) Given the Velicer and associates criterion, it is perhaps surprising that they still report no important differences between common factor analysis and component analysis, thus suggesting that common factor analysis is robust even to a component based criterion. And common factor analysis was also better in reproducing the original population values in Snook and Gorsuch. So common factor analysis is satisfactory with a component or a population criterion, although component analysis is not.

Another difference between the Velicer and associates’ articles and Snook and Gorsuch (1989) is that the former did not test for bias in the loadings. The criterion they used — labeled $g$ — was the average square of the difference between the observed loadings and the principal component loadings from the population correlations. Snook and Gorsuch likewise used the square but also used the raw difference between the observed loadings and the population loadings as an indicator of possible bias. We labeled a systematic bias as inflated just as in other Monte Carlo studies: a value which is systematically higher than the population value. Deflated would be values systematically lower than the population value. By this criterion common factor loadings were unbiased and component loadings were inflated.

Note here that there is a parallel between component analysis and item-total correlations, on the one hand, and common factor analysis and item-remainder correlations on the other hand. The former two can include in the correlation the variable’s own error whereas the latter two reduce or exclude that error; hence common factor and item-remainder correlations are generally lower than component and item-total correlations, which are inflated due to correlated error.

4. Numerous people have reported Heywood cases, that is, inappropriate communalities resulting from iterating the communality estimates. The reason is clear: iteration is wrong (Gorsuch, 1974, 1983). The reason iteration began is because iteration is mathematically correct in many situations which we know through mathematical proofs. Also upon examining the estimates after the first two or three iterations — particularly when population values are unknown — it seems that they are converging. Humphreys and Taber (1973) show, however, that the convergence can be diverging from the true value. What was said in 1983 still holds true: “No proof has yet been presented to show that they either must converge or that the value to which they converge is either the
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theoretical communality or less than the variable’s reliability” (Gorsuch, 1983, p. 107, boldface in original). When a procedure for which there is no proof leads to erroneous results, the appropriate response is to drop the procedure.¹

The question of why iteration is a well-established belief when there is no mathematical proof for and negative evidence against iteration is, of course, a psychological problem rather than a mathematical one. Perhaps it may be best viewed as the human tendency to have faith in generalizations. The fact of improper solutions in common factor analysis is therefore a problem of using a technique that seems good but is wrong.

Other procedures do give correct communality estimates when correct is defined as not significantly different from the population communalities. One procedure was noted by Gorsuch (1974, 1983): using the squared multiple correlation plus two to three iterations. Snook and Gorsuch (1989) cross-validated that this procedure produces communality estimates which are not significantly different from population values. (With a large number of variables, using the highest correlation in a column appears satisfactory but Monte Carlo studies to confirm this recommendation are needed.)

5. Velicer and Jackson state that it is difficult to calculate common factor solutions due to time and size of computer. Of course, when using a correct communality estimation procedure rather than iterating, the time for common factor analysis drops by one to two orders of magnitude, making it only slightly longer than extracting principal components (and the same if one uses the highest correlation in the column without an iteration).

With regard to the number of variables that can be factored, Gorsuch and Dreger (1979) provide a program for factoring 500 variables by common factor techniques. I currently have just completed a common factor analysis of 320 items on my personal computer using only an 8088 processor and 500K of memory; it is very slow but doable.

Hence common factor analysis is readily done.

6. Most common factor procedures estimate rather than calculate factor scores. This, however, is at the choice of the person doing the common factor analysis. I define a common factor analysis as any factor analysis which includes error explicitly in its model, so image analysis is a common factor analysis which results in directly calculated factor scores. Hence if factor score indeterminacy is a problem for you, use image analysis. (If those urging

¹ I feel somewhat responsible for the continued misplaced faith in iterating communalities because I recognized the problem with the first edition of my text but failed to see the need to take further action. I thought that a discussion in my textbook would be sufficient. It now appears that a series of articles would have had more impact so that the discussion could have been farther along than it is today.
component analysis would be willing to compromise by using image analysis as the only appropriate factoring technique, I shall be happy to agree.

Recently the factor score problem for common factor analysis has been reduced by a new procedure which computes the correlations between the factors and variables not in a factor analysis without using implicitly or explicitly factor scores (Gorsuch, 1989). Hence by either using image analysis or the new technique, factor scores are a minor issue, as Velicer and Jackson (1990) note.

7. In both mathematics and science, elegance is a desirable property. The definition of elegance follows William of Occam's razor principle: the most direct and robust solution is the best (i.e., shave off the unnecessary). For example, Einsteinian relativity theory is more elegant than Newtonian theory because Newtonian theory is a special case which leaves many "loose ends" hanging which relativity theory includes. In the same manner, the debate before World War II about whether to use the average deviation or the standard deviation was resolved in favor of the standard deviation because of its greater elegance: although average deviation was easier to compute and simpler to explain, standard deviation fits into the broader statistical models of, for example, variance and correlation.

Note that both these cases of illustrating elegance — relativity theory and standard deviation — are situations where, as with component analysis, the less elegant solution is seen at first to be easier to understand and calculate. However, that solution failed to fit into the broader models of physics or statistics and so, from a broader model perspective, added complexity rather than simplification and so was dropped. Common factor analysis when derived as in Gorsuch (1974, 1983) uses a unified multivariate — or unimult — model with error of which canonical analysis, MANCOVA, ANOVA, multiple regression, and confirmatory factor analysis are special cases.

History and Paradigms

Paradigms often influence what techniques are used in addition to the logic of those techniques, and history affects paradigms. Hence some historical comments regarding the components versus common factors controversy may be useful.

The widespread use of extracting principal components with latent roots greater than one followed by Varimax originated with Henry Kaiser at the University of Illinois. He provided "Little Jiffy" because it could be run on the first generator computer Illiac by people with no training and provide some results for a term or class paper (Kaiser, personal communication). He has, of course, attempted to reduce the use of this component-based analysis by
comments at conventions and professional meetings that I heard him make as well as by providing alternatives (e.g., Kaiser, 1970). The easy programmability of “Little Jiffy” appears to be why many use a technique rejected by its originator.

Part of the reason for components’ popularity underlies Jum Nunnally’s recommendation of it in his textbooks (1968, 1978). Indeed, I suspect that lack of sophistication among programmers and Nunnally’s texts are primary reasons that components are widely used, and so understanding the historical situation within which Nunnally developed his stance may be useful. I feel I have some personal knowledge regarding his opinions because (a) I was the ABD who critiqued every page of the first edition of his text and calculated all the examples, (b) I took over the graduate course at Vanderbilt when he published his text and thus used his text in his department, and (c) Jum was the major outside reader who gave page by page comments on my factor analysis text draft.

First we must remember that the popularity of components with people such as Nunnally occurred during the first and second generation computers of the 1950s and 1960s. Jum’s factor analytic work was almost entirely within this period. During that time the computers had, literally, no more memory than an Apple II and speed was also a major problem. Thus the fact that component analysis could be done in that environment where iterated common factor analysis — we not yet knowing any better than to iterate — was truly time-consuming and impossible, led to people such as Jum doing component analyses. Jum further argued for component analysis as a teaching technique using equations students could readily calculate, but he also taught centroid analysis for the same reason. That was, of course, before we realized how naturally common factor analysis falls out of a generalized multivariate model.

Nunnally’s conclusion is that with many variables the results are much the same, a position which I learned from him and developed also (Gorsuch, 1974, 1983, 1988). The difference was how we defined many, with Jum drawing the line at approximately 20 variables (Nunnally, 1978, p. 419f) and my suggesting 30 variables. (Snook and Gorsuch, 1989, show that we should have drawn that line at 40 variables.)

What did Jum recommend when components and common factors might differ? Common factor analysis: “There are several sensible choices for the experimenter: (1) Place SMCs [i.e., squared multiple correlations] in the diagonal spaces before the analysis, (2) Employ minimum residual analysis, or (3) Use image analysis” (Nunnally, 1978, p. 420). Jum recommended these whenever one had a situation where components and common factors might differ. Hence Nunnally’s functional model was common factor analysis, and component analysis was only acceptable insomuch as it produced almost identical results to common factor analysis. Where they diverged, Jum
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Nunnally recommended common factor analysis. For that reason I have found it amusing that some see Jum as a proponent for component analysis as if he and I had radically different positions. Perhaps we should find encouragement in this for rapprochement among those currently in dialogue regarding components and common factors.

Conclusion

Common factor analysis should be routinely applied as the standard analysis because it recognizes we have error in our variables, gives unbiased instead of inflated loadings, and is more elegant as a part of the standard model used in univariate and multivariate analysis. The continued use of components is primarily the result of decisions made when there were problems computing common factor analysis which no longer exist and the continuation of its being a ready default on computer programs designed during an earlier era.

References


